(Pages : 4) **2454**

Reg. No. : .....

Name : .....

Fourth Semester B.Sc. Degree Examination, June 2014
First Degree Programme under CBCSS
Complementary Course for Physics
MM 1431.1 : MATHEMATICS – IV (Complex Analysis, Fourier Series and Fourier Transforms)

Time : 3 Hours

Max. Weights : 30

SECTION – A

All questions are **compulsory** in this Section. There are **16** questions in **4** bunches. **Each** bunch carries **1** Weight.

I. 1) Find the value of $e^{(2 \pm 3\pi i)}$.

2) What is the principal branch of log (–i) ?

3) Define cos z in terms of e^{iz}.

4) Write down the imaginary part of z^2.

II. 5) Give an example of a harmonic function.

6) Check the analyticity of f(z) = xy + iy.

7) Calculate the value of the integral $\int_0^{\pi/6} e^{i2t}\, dt$.

8) State Cauchy's-Residue theorem.

III. 9) What is the formula for finding the radius of convergence of a power series $\sum_{n=1}^{\infty} a_n Z^n$?

10) What are the singular points of $f(z) = \dfrac{z}{(z-1)^2 (z-2)}$?

P.T.O.

All Saints' College Trivandrum

11) Find the residue of $f(z) = \frac{z}{z^2 + 1}$ at $z = +i$.

12) Define Fourier series.

IV. 13) Write down the coefficient of Fourier sine series.

14) Find the value of the Fourier coefficient a_0 for the function $f(x) = |x|$ in $-\pi \leq x \leq \pi$.

15) State convergence theorem in Fourier series.

16) Define Fourier transform. **(4×1=4 Weights)**

SECTION – B

Answer **any 8** questions among the questions **17** to **28**. **Each** question carries **1** weight.

17) Prove that analytic function is constant if its real part is constant.

18) Show that $u = y^3 - 3x^2y$ is harmonic.

19) Find imaginary part of the analytic function whose real Part is $u = x^2 - y^2$.

20) Verify Cauchy-Reimann equations for the function $f(z) = \sin z$.

21) Calculate the value of the integral $\int_C \frac{1}{z-2} dz$, where C is the circle $|z| = 1$.

22) Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{z-3} dz$, where C is the circle $|z - 2| = 2$.

23) Find the radius of convergence of $\sum_{n=1}^{\infty} \left(\frac{1}{np} \right) z^n$.

24) Expand $\frac{1}{1-z}$ in $0 < |z| < 1$.

25) Obtain the Fourier coefficient b_n for the function $f(x) = e^{-x}$ in $0 < x < 2\pi$.

All Saints' College Trivandrum

26) Find the half range cosine series for $f(x) = x$ in $0 < x < \pi$.

27) Explain Fourier sine and cosine integrals.

28) State sufficient condition for the existence of the Fourier transform. **(8×1=8 Weights)**

SECTION – C

Answer **any 5** questions from among the questions **29** to **36**. **Each** question carries **2** weights.

29) Find the analytic function whose real part is $e^x (x \cos y - y \sin y)$ and which takes the value e at $z = 1$.

30) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin, even though C-R equations are satisfied at that point.

31) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along $y = x^2$.

32) Evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} dz$, where C is the circle $|z - 1| = 1$.

33) Find the sum of the residues of the function $f(z) = \frac{2z - 1}{z(z + 1)(z - 2)}$ at the poles inside the circle $|z| = 2$.

34) Obtain the Fourier series for $f(x) = x^2$ in $[-\pi, \pi]$.

35) Find the half-range sine series for

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$$

36) Obtain Fourier cosine integral of $f(x) = e^{-kx}$, $(k > 0, x > 0)$. **(5×2=10 Weights)**

All Saints' College Trivandrum

SECTION – D

Answer **any 2** questions from among the questions **37** to **39**. **Each** question carries **4** weights.

37) If $u - v = e^x (\cos y - \sin y)$ and $f(z) = u + iv$ is analytic, then find $f(z)$ in terms of z.

38) Expand $\frac{1}{z(z^2 - 3z + 2)}$

i) $1 < |z| < 2$

ii) $|z| > 2$.

39) Find Fourier of the function

$$f(x) = \begin{cases} -1 , & -\pi < x < 0 \\ 0 , & x = 0 \\ 1 , & 0 < x < \pi \end{cases}$$

and deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \infty = \frac{\pi}{4}.$$

(2×4=8 Weights)

All Saints' College Trivandrum